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ROBUST OPTIMIZATION IN OPERATIONAL RISK: A STUDY OF THE JOINT PLATFORM ALLOCATION TOOL

by

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June 2014

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The Joint Platform Allocation Tool (JPAT) is a tool currently used to inform Army decision makers on resource management, procurement, and operational employment of Army aerial intelligence, surveillance, and reconnaissance (ISR) assets. The tool is modeled and implemented using point estimates for input data on future resource, equipment capability, and employment demand. This research expands the capability of the JPAT to account for uncertainty and changes in those parameters that bear on the overall operational risk of the Army's ISR mission: uncertain and changing future budgets, and uncertainty and unpredictability of future operational demands for ISR assets. Techniques of robust optimization are explored and applied to JPAT, and results and methodology are shown to be applicable to other operational areas.

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PLATFORM ALLOCATION TOOL

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The Joint Platform Allocation Tool (JPAT) is a tool currently used to inform Army deci-sion makers on resource management, procurement, and operational employment of Army aerial intelligence, surveillance, and reconnaissance (ISR) assets. The tool is modeled and implemented using point estimates for input data on future resource, equipment capability, and employment demand. This research expands the capability of the JPAT to account for uncertainty and changes in those parameters that bear on the overall operational risk of the Army's ISR mission: uncertain and changing future budgets, and uncertainty and unpre-dictability of future operational demands for ISR assets. Techniques of robust optimization are explored and applied to JPAT, and results and methodology are shown to be applicable to other operational areas.

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List of Acronyms and Abbreviations

DPG defense planning guidance

GAMS General Algebraic Modeling System

ICDT Integrated Capabilities Development Team

ISR intelligence, surveillance, and reconnaissance

JPAT Joint Platform Allocation Tool

MIP Mixed-Integer Program

OSD Office of the Secretary of Defense

POM Program Objective Memorandum

RO robust optimization

SP stochastic programming

TRAC TRADOC Research and Analysis Center

TRADOC U.S. Army Training and Doctrine Command

UAV unmanned aerial vehicle

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Executive Summary

The Joint Platform Allocation Tool (JPAT) is a tool currently used to inform Army decision makers on resource assignment, procurement, and operational employment of Army aerial intelligence, surveillance, and reconnaissance (ISR) assets. The tool is modeled and implemented using point estimates for input data on future resource, equipment capability, and employment demand, and does not consider the uncertainty and variation inherent in such data. This research seeks to account for such uncertainty and sensitivity in the formulation of such models to help inform better decisions and give the decision maker a sense of the risk of a given course of action.

An accepted and developed way to approach such problems is through stochastic programming. However, most stochastic programming methods are viable only on relatively small problems. The complexity and computational cost of applying stochastic programming to large optimization problems can prove prohibitively expensive.

This research takes a different approach. We explore robust optimization methods and techniques of relatively recent development and apply them to the JPAT. For most classes of problems, their robust counterparts are of the same class and computational cost. In this way uncertainty can be accounted for in the optimization and decision making process without being prohibitively expensive computationally. In this thesis, robust counterparts to the JPAT baseline are formulated in ways that apply variation to three of the models input values: operating budgets, mission priorities, and demand hours per mission. Using a test data set, the solution of the baseline is compared to its robust counterparts' for different scenarios and analyzed in its sensitivity in term of utility and computational cost.

This thesis focuses on one tool in the Army's analysis arsenal. However, all military decision making involves some level of uncertainty and risk. The robust optimization methodologies and the formulation techniques developed here and applied to the JPAT may be applied to many other types and classes of problems. Doing so provides a direct and computationally efficient method of accounting for risk and better informing military decision making.

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CHAPTER 1:

Introduction

This chapter places the Joint Platform Allocation Tool (JPAT) in its operational context.

1.1 Operational Background

In 2011, in the context of mission changes in Iraq and Afghanistan and Office of the Secretary of Defense (OSD) emerging guidance on the future strategic environment, the Army began assessing its future resource strategies for aerial intelligence, surveillance, and reconnaissance (ISR) requirements (in the form of unmanned aerial vehicles (UAVs)). In addition to planning for future aerial ISR needs and emerging technologies, there was a legacy of quick-reaction capabilities accrued from the campaigns in Iraq and Afghanistan that would require some sort of disposition: maintain and expand current ISR systems, or retire and replace them with something more effective, affordable, etc. In October 2011, the Army TRADOC commander established the Aerial R&S Integrated Capabilities Development Team (ICDT) to assist Army decision makers in determining the future resource strategy for the Army's aerial ISR requirements. Defense Planning Guidance (DPG) and the Army's vision of the strategic environment of 2020 (Army 2020 [1] and Army Intel 2020 [2]) influenced the work of the ICDT, and the Program Objective Memorandum (POM) 14-18 was used as an analytical baseline for their efforts.

One of the key objectives of the ICDT was to inform recommendations and decisions related to ISR requirements of the POM 15-19. To meet this objective, the ICDT sought to answer four key study issues: 1) What is the range of intelligence demands over time that Army aerial ISR systems will have to fulfill? 2) What mixes of Army aerial ISR assets best satisfy the range of demands given joint and national ISR resource contributions? 3) What are the resource implications of a given aerial ISR resource mix? 4) What mix of Army aerial ISR assets is most cost-effective? The ICDT developed the JPAT to inform answers to the above study issues. The objective of the JPAT model is to satisfy a 12-year intelligence demand signal by determining what systems should be procured and where they should be fielded, where current systems should be repositioned, how system configurations should be assigned to meet mission demands, and when and what systems should

be retired from the inventory.

1.2   Technical Background

The Joint Platform Allocation Tool (JPAT) is a mathematical model implemented in GAMS. The formulation of the JPAT by Craparo et al. [3] is "currently used to evaluate the strategic implications of cost, sensor performance, mission requirements, and production timelines to produce optimal procurement and assignment schedule of aerial reconnaissance and surveillance assets." [4] The model is formulated as a mixed integer program that maximizes hours of mission demands met and is solved sequentially over a rolling horizon time period. This is due to the computational cost of reaching a solution in the current formulation. Because the model looks 12 years into the future (and each year is divided into 12 months), and allowing for the rolling horizon, sensitivity analysis on the implications of uncertainty or change on future parameters cannot easily be performed.

The inputs to the JPAT model are point estimates, many of which represent uncertain future values. Formulating the inputs in this way is common practice, but can often misrepresent problems and the potential risk of a given model's output. This thesis demonstrates robust optimization (RO) as an alternative to the point estimate approach.

Robust optimization techniques can be applied to the current formulation of JPAT and can provide insight to decision makers on the effects that uncertainty can have on the optimal solution given certain boundaries of uncertain parameters or constraints (future budget cuts, or unforeseen mission demands, for example). This research focuses on the JPAT, but most optimization models are likely candidates for the application of robust optimization techniques. This research seeks to demonstrate how robust optimization can account for uncertainty and sensitivity in such models to inform better decisions. The techniques explored here help analysts consider the inputs to their models as ranges, rather than point estimates. This is not only more realistic, but it also serves to focus the mind on the impact of that variability in an operational context.

CHAPTER 2:
Literature Review

In this chapter, the development of the Joint Platform Allocation Tool is discussed in terms of its operational context. Robust Optimization theory are discussed, and methods and techniques are discussed and related to applicable features of the JPAT formulation.

2.1 Robust Optimization Efforts

There are many studies of robust optimization widely available. Its attractiveness as a field of study is based on the premise that solutions to optimization problems are sensitive to perturbations of input parameters that can often make the solution suboptimal or infeasible [5]. This concept has been addressed in several ways since the 1950s and the early developments of decision theory and extensively developed in the field of stochastic optimization.

Unlike stochastic optimization, which assumes a probabilistic distribution of the uncertainty in an optimization problem, robust optimization models uncertainty as deterministic and set based. Ben-Tal and Nemirovski [6] lay much of the foundation for the theory of robust optimization. They studied convex optimization problems for which the input data is not necessarily specified and belongs to a set of uncertainty that can be modeled as ellipsoidal. They show that the robust formulation to a wide variety of common convex optimization classes is as tractable computationally as the underlying non-robust formulation. Bertsimas et al. [7] claim that constructing a deterministic uncertainty set over which any realization in that set yields a feasible solution is preferable to a stochastic approach because it tends to conserve computational tractability with only a small trade-off in optimality.

Bertsimas et al. [7] show that the robust approach to optimization has several advantages. First, with careful consideration of the model of the uncertainty set, robust formulations of a general optimization problem remain tractable, particularly well-known classes of optimization problems such as linear programming and mixed-integer programming (to which class the JPAT optimization belongs). Additionally, the robust formulation of an optimiza-

tion problem allows the modeler to make trade-offs between performance and robustness in a concept Bertsimas [7] calls 'conservativeness,' in which the design of the deterministic uncertainty set can be constructed to reflect a probabilistic distribution a priori optimization, or to restrict the uncertainty set or solution in a desired manner.

Many parts of the JPAT formulation are related to classic supply-chain or inventory control problems. For example, constraints that deal with the on-hand inventory of ISR equipment at various locations at a given time are common to both situations. In the case of the JPAT model, there is uncertainty in the mission demand for the inventory of ISR assets as well as uncertainty on the size of the "inventory" of budget dollars that are required to meet demands. Bertsimas and Thiele [8] show that robust approach to modeling these types of uncertainty are numerically tractable and do not increase the computational complexity of the nominal problem. They do this by allowing a scaled deviation of an uncertain parameter from its nominal value in the formulation of the linear problem, which has a robust linear counterpart (with a proof based on strong duality). One of the benefits of this approach is that it allows a scaled approach to the conservativeness of a solution by allowing a constraint on the amount of uncertainty expressed by the uncertain parameters; that is, "it allows tradeoff between performance and robustness."

Ben-Tal et al. [9] also address the problem of multi-period asset allocation and develop a robust optimization counterpart to a classic portfolio problem. They show that the portfolio policy of the robust counterpart is significantly more robust than nominal or stochastic modeling policy in simulations, and it is comparable in optimality. They also show that the computational requirement of the robust approach is significantly less than the multistage stochastic approach. These concepts can have similar applications to the JPAT formulation where there are multiple types of assets that can be applied over many time periods that affect the objective value of the problem.

CHAPTER 3:
Methodology

This chapter begins with the presentation of the baseline JPAT formulation presented in section 3.1. Section 3.2 presents stochastic programming and its shortcomings as a possible method of exploring the effect of uncertainty in the formulation of the JPAT and why it would be of limited value for the purposes of this thesis. Several robust counterparts of the JPAT formulation appear in section 3.3. Finally, section 3.4 presents challenges posed for pessimistic scenario generation and robust counterparts formulated to overcome these difficulties.

3.1   Baseline JPAT Formulation

For research purposes of this thesis, an unclassified version of JPAT was recovered from the classified environment. The JPAT formulation is implemented in General Algebraic Modeling System (GAMS) as a mixed-integer program (MIP). The formulation employs a rolling horizon time frame to account for sequential decision making and to make the MIP tractable for each time frame. In the JPAT formulation the rolling time frame is one year, with time steps of one month. In this chapter we follow Naval Postgraduate School Technical Report NPS-OR-13-004 for the problem formulation [3].

Indices and Sets

$y, y' \in Y$	System y in set of all possible systems Y.
$c \in C$	Configuration c in set of all possible configurations C.
$e \in E$	Equipment item e (to include platforms and payloads) in set of all considered equipment E.
$(t, y, l, l') \in GP$	Identifies systems y eligible to transfer from location l to location l' at time t.
$(y, y') \in REP$	Identifies the system y' replacing a retiring system y.
$l, l' \in L$	Location l and alias l' in set of all possible locations L.
$t, t' \in TIME$	Time step t and alias t' in set of all possible time steps T.

$m \in M$	Specific mission demand m in set of all mission demands M (later organized in set for time and place).
$i \in I$	INT (intelligence) types I.
$r \in R$	Iterations in the rolling horizon model.
$t \in T(r) \subseteq TIME$	Time steps considered in an iteration r.
$t \in N \subseteq TIME$	Set of time steps at the beginning of a fiscal year.
$M(l)$	Set of mission demands residing in location l.
$l(m)$	Location of mission demand m (each mission demand resides in exactly one location).

Input Data

$iq_{e,l}$	Initial quantity of equipment e in location l at time 0. [items]
$d_{t,m}$	Number of times mission demand m is present at time t. [occurrences]
$ok_{m,i,c}$	Number between 0 and 1 indicating the ability of configuration c to fulfill requirement type i in mission demand m. [unitless]
omc_e	Operation and maintenance (O&M) cost per month for equipment e. [$M]
pc_y	Procurement cost for system y. [$M]
rc_y	Retirement cost for system y. [$M]
$b_{t,y}$	Maximum budget for system y at time t. [$M]
$pr_{t,y}$	Maximum production rate of system y at time t. [items]
p_m	Number between 0 and 1 indicating the importance of mission demand m. [unitless]
$ec_{c,e}$	Number of equipment e in configuration c. [items]
$es_{y,e}$	Number of equipment e in system y. [items]
he_e	Hours available for transport and missions per time period for equipment e, accounts for regular maintenance hours, etc. [hours]
hm_m	Hours required to perform mission demand m, not including equipment-specific setup and take down time. [hours]
$hi_{m,i}$	Hours required for requirement type i in mission demand m. [hours]
su_e	Hours to set up, take down, and maintain equipment e per assignment. [hours]

$ht_{e,y,l,l'}$	Hours required to transfer equipment e as part of system y from location l to location l'. [hours]
$sr_{m,c}$	Sorties required in order for configuration c to fully complete mission demand m. [sorties]
$maxdist_{t,y}$	Maximum number of system y that can be distributed as of time t. [items]
$mr_{t,y}$	Total number of system y that must be retired by time t. [items]
$initial_y$	Number of system y initially in theater. [items]

Decision Variables

$G_{t,y,l,l'}$	Number of system y transferring from location l to location l' at time t.
$Z_{t,y,l}$	Number of system y retiring from location l at time t.
$D_{t,y,l}$	Number of system y distributed to location l at time t.
$P_{t,c,l}$	Binary variable equal to 1 if sufficient equipment is present to create configuration c at time t in location l; 0 otherwise.
$X_{t,m,c,i}$	Number of hours configuration c is assigned to mission requirement type i for mission demand m at time t.
$S_{t,m,c}$	Number of sorties flown by configuration c against mission demand m at time t.
$Q_{t,e,l}$	Quantity of equipment e present in location l at time t.
B_t	Budget rolled over from previous time period at time t.

Formulation

For readability and convenience, the formulation is shown in Figure 3.1. The following description of the formulation is taken from Naval Postgraduate School Technical Report NPS-OR-13-004 by Craparo et al. [3].

The objective function (1) maximizes the weighted mission demand coverage, weighted by mission demand priority and configuration performance. Constraint set (2) ensures that intelligence requirements are not oversatisfied by the assigned configurations. Constraint sets (3-4) maintain a record of the quantity of each equipment type available in each location, beginning with the initial quantity (4) and updating the quantity based on system

procurements, retirements, and transfers in subsequent time steps (3).

Constraint sets (5-8) ensure that configurations are employed appropriately based on equipment availability. Constraint set (5) forces $P_{t,c,l}$ to take on a value of zero if any piece of equipment require to construct configuration c is not present in a sufficient quantity in location l at time empht; otherwise, $P_{t,c,l}$ is allowed to take on a value of one. Constraint set (6) uses the variables $P_{t,c,l}$ to control the number of sorties flown by configuration c: if $P_{t,c,l} = 0$, then configuration c cannot fly any sorties against any mission demands in location l at time t. Otherwise, configuration c can fly any number of sorties so long as it does not exceed the number of sorties required to completely satisfy the mission demand. Constraint set (7) ensures that the time spent covering intelligence requirements is appropriate given the number of sorties flown. Finally, constraint set (8) ensures that the hours spent fulfilling mission demands and transferring from one location to another do not exceed the"pool" of hours available for each equipment type.

Constraint sets (9-11) ensure that budgetary limitations are observed. Constraint set (9) calculates the monthly budget rollover B_t while accounting for equipment maintenance, system procurement, and system retirement costs. Because B_t is a nonnegative variable, constraint set (9) ensures that the available budget is not exceeded on months that do not mark the beginning of a fiscal year. Likewise, constraint set (10) performs this function for months that do mark the beginning of a fiscal year, while constraint set (11) sets B_t to zero for months at the beginning of a fiscal year.

Constraint sets (12-13) control distribution and retirement of systems. Constraint set (12) ensures that the total number of system y distributed as of time t does not exceed the limits posed by system production rates and fielding restrictions. Constraint set (13) ensures that any system y' that "upgrades" a system y is not distributed until its predecessor y is retired. Finally, constraint sets (14-21) declare variable types.

3.2   Multi-stage Stochastic Programming Discussion

In the baseline formulation, several input parameters actually represent decisions that are made before the initial time-step, and do not change even though they should rely on data unavailable after the first time-step. For instance, the budget in month 32 is always the same, even though events and decisions prior to month 32 may drive the decision maker to

increase or decrease the budget accordingly.

A multi-stage stochastic programming model lends itself to this type of decision making. While a multi-stage stochastic programming approach would be a good representation of this type of model and decision making, it comes at a huge computational cost. The multi-stage programming approach can provide a great deal of information and optimal solution space for the initial stage of the process, but becomes a computationally prohibitive method for a full run of the optimization model [9].

For comparison purposes, it is reasonable to compare the RO formulation to a multi-stage stochastic programing formulation small, controlled test set of data. In recognition of the adequacy of the stochastic programming approach, it is worth testing the RO formulation against this to measure differences in the objective values and solutions among them, to see the small cost to the solution compared to the huge savings in computational tractability.

The parameters that we bring into variation (budgets, priorities, mission demands), should be handled differently in the different stages of a multi-stage stochastic approach. It makes sense that budgets for the current and a subsequent few time-steps are known. For this, making the realizations of these budgets in the first stage of the stochastic program would make sense. The actual priority of a mission or the demand hours for a given mission (as opposed to what is expected or planned from the SME perspective) would not likely be known until execution of the mission (or even until after it). Having the realizations of these parameters in the second step of the stochastic program would be an appropriate way to represent this.

For our purposes, the multi-stage stochastic program can take two forms, either a sampling-based or non-sampling-based approach.

In the sampling-based approach, the initial stage consists of choosing a candidate package of UAV assignments. In the second stage we estimate the expected value of the objective function with respect to the variables that are treated as random. Hence, the second stage consists of sampling the parameter (in this example, the hours of demand) from a defined underlying distribution and making optimal assignments accordingly; this is repeated a number of times (up to a prescribed sample size), and produces the estimator by averag-

ing the values of the objective function. In two dimensions this approach can be easily visualized as estimating the maximum value of a curve.

Basic formulation

$$\max_{P,G,Z,D,S,X,B,Q} z = \sum_{(t,m,c,i):t\in T(r),d_{t,m}>0.hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,i,c}}{\sum_{i'} hi_{m,i'}}$$

s.t.

$$\sum_{c:ok_{m,i,c}>0} X_{t,m,i,c} \leq hi_{m,i}d_{t,m} \quad \forall t \in T(r),e,l:t>1$$

$$...$$

Initial stage variables: G,P,Z,D,B,Q
Second stage variables: X,S
Problem:

$$\max_{G,P,Z,D,B,Q} E_\omega[\max_{X,S} \sum_{(t,m,c,i):t\in T(r),d_{t,m}>0.hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,i,c}}{\sum_{i'} hi_{m,i',\omega_j}}]$$

Random search approach:
For loop over 100 iterations (outer loop):
First stage: draw random G,P,Z,D,B,Q from their domain.
For loop over 100 iterations (inner loop, second stage random):

$$\text{draw } hi_{m,i',\omega_j}$$

then

$$\max_{X,S} z_j = \sum_{(t,m,c,i):t\in T(r),d_{t,m}>0.hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,i,c}}{\sum_{i'} hi_{m,i',\omega_j}}$$

s.t.

$$\sum_{c:ok_{m,i,c}>0} X_{t,m,i,c} \leq hi_{m,i}d_{t,m} \quad \forall t \in T(r),e,l:t>1$$

$$...$$

End for loop.
Compute average $\frac{1}{100}\sum_j z_j$

The optimal G, P, Z, D, B, Q of the 100 random realizations from the outer loop is one with largest $\frac{1}{100} \sum_j z_j$

The non-sampling method takes a different approach. In it, we assume a probability distribution with finite support for the random parameter. As in the sampling-based approach, the first stage consists of finding a candidate package of UAV assignments. In the second stage, we solve for the optimal package of UAV assignments for each point of mass of the random parameter, and then compute the expected value of the objective function by weighting each second-stage objective function value by the probability mass at each support point. This approach is deterministic, and hence is not hampered by sampling error considerations. However, the number of second-stage problems that need to be solved equals the points of mass of the random parameter. In particular, when several parameters are treated as random, the number of second-stage problems equals the product of the cardinality of the support of each random parameter, meaning that the number of second stage problems grows exponentially with the number of random parameters.

Basic formulation

$$\max_{P,G,Z,D,S,X,B,Q} z = \sum_{(t,m,c,i):t \in T(r), d_{t,m}>0.hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,i,c}}{\sum_{i'} hi_{m,i'}}$$

s.t.

$$\sum_{c:ok_{m,i,c}>0} X_{t,m,i,c} \leq hi_{m,i} d_{t,m} \quad \forall t \in T(r), e, l : t > 1$$

$$\ldots$$

Initial stage variables: G, P, Z, D, B, Q
Second stage variables: S, X
Problem:

$$\max_{G,P,Z,D,B,Q} E_\omega [\max_{S,X} \sum_{(t,m,c,i):t \in T(r), d_{t,m}>0.hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,i,c}}{\sum_{i'} hi_{m,i',\omega_j}}]$$

s.t.

$$\sum_{c:ok_{m,i,c}>0} X_{t,m,i,c} \leq hi_{m,i} d_{t,m} \quad \forall t \in T(r), e, l : t > 1$$

$$\ldots$$

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Expected value approach:

First stage: draw random G, P, Z, D, B, Q from their domain.

For each point of probability mass in pmf (hi_{m,i',ω_j}):

$$\text{calculate Expected Value: } hi_{m,i',\omega_j} * P(hi_{m,i',\omega_j})$$

then

$$z_j = P(hi_{m,i',\omega_j})[max_{S,X} \sum_{(t,m,c,i):t \in T(r), d_{t,m} > 0.hi_{m,i} > 0} p_m ok_{m,i,c} \frac{X_{t,m,i,c}}{\sum_{i'} hi_{m,i',\omega_j}}]$$

End for loop.

Compute E.V. $\sum_j z_j * P(hi_{m,i',\omega_j})$

The optimal G, P, Z, D, B, Q is one with largest $\frac{1}{100} \sum_j z_j$

3.2.1 Computational Cost

For the sampling-based approach, the number of stage 1 problems equals the number of sample points. In one extreme, a small number of sample points results in a relatively low cost to "optimize," but its optimal solution and value of the objective function are likely off from their optimal values, because the variance is high. In the other extreme, allowing for a large number of sample points spends most of the computational budget solving large number of problems, each of which is necessarily stopped earlier than in the first case, meaning that the solution and value of the objective function are more biased. This is known in the literature as the *variance-bias* trade off.

The computational cost of the deterministic equivalent approach is likewise prohibitively expensive. In this case the number of stage 2 problems equals the product of the cardinality of the support of each random parameter, which grows exponentially with the number of random parameters.

These considerations suggest that stochastic programming methods are unsuitable for this situation. A mixed-integer program like the JPAT is already computationally costly: a stochastic reformulation of the model would only compound the computational difficulty and would be NP-hard. This thesis does not seek to create such a formulation. A constraint of the study was to maintain at least the same computation difficulty class as the baseline,

and would have been of limited value to the sponsor and future projects.

3.3 Points of Entry

Three parameters in the JPAT formulation are identified as ideal candidates for robust optimization (RO) reformulation. The first are the monthly budget constraints. The second are the priority value of particular missions. The third are the monthly mission demands (more specifically, the number of hours required for a particular mission in a given month). Each of these are point estimates based on solicitation from subject matter experts (SME), and are logical candidates for the modeling of uncertainty in the model. Each of these parameters appear in the formulation in a unique way (mission priority appears only in the objective function, for example) and present unique challenges and insights.

The monthly budget amounts were a good entry point for the introduction of variability and uncertainty into the JPAT formulation for several reasons. It is conceptually easy to understand that budgets are not always certain, especially in the current fiscal environment. Budget decisions are among the most important types of decisions that leaders must deal with, so there is great benefit to exploring the implications of variability on the budgets to gain insight for decision makers. The budget for each time period appears in the formulation in the right hand side of several constraints.

The priority of a mission weights the value of its associated mission in the objective value. These are not necessarily normalized (that is, they do not sum up to 1). The priorities were considered for reformulation from a parameter to a RO variable because they strictly appear in the objective function of the baseline formulation. This provides a unique modeling challenge as compared to the budget reformulation and has the potential to provide much insight for the decision maker, especially if they can be reformulated in a way that they values are normalized in way that provides more inherent meaning.

Monthly mission demands appear as 3-dimensional parameters in the baseline formulation, presented as a triple of time, location, and intelligence type. They appear in constraints as well as part of the objective function. Conceptually, uncertainty on the mission demand is easy to understand as one can imagine, for example, that a UAV platform on a specific mission may be needed to stay on station longer to exploit a developing situation. In any case, the realization of the mission demands cannot be known with certainty ahead of time.

In every case of the reformulations above, we assume the nominal parameter values of the baseline formulation that are solicited from SMEs are accurate on average, or there is no systemic bias. However, we may assume that for any given nominal value, the actual value may vary by a certain percentage. So for example, to incorporate demand variability, we allow a range of possible hours for a given intelligence type in a give time and location about the nominal value given in the baseline, and when a demand is met two things happen: A value is accrued in the objective function, and resources are consumed (in the constraints).

3.3.1 Budget

Regarding the budget, there are a number of scenarios for exploration: optimistic or pessimistic, and with or without transfers. As stated previously the baseline formulation has the budget parameters appear only in the constraints:

$$B_t = B_{t-1} + \sum_y b_{t,y} - \sum_{y,l} \left(pc_y D_{t,y,l} + rc_y Z_{t,y,l} \right) - \sum_{e,l} omc_e Q_{t,e,l}$$

$$\sum_{y,l} pc_y D_{t,y,l} + \sum_{y,l} rc_y Z_{t,y,l} + \sum_{e,l} omc_e Q_{t,e,l} \leq \sum_y b_{t,y}$$

To formulate the robust counterpart for the budget parameters, the vector of nominal budget values remains $b_{t,y}$, around which the new RO variables $BU_{t,y}$ are allowed range. A parameter that controls the range about the nominal value is allowed to range is introduced, k. For example, allowing for a 10% variation in either direction, we would have $k = 0.10$. To control the total amount of variation across all constraints the budget of variability value Γ is introduced and can be defined as the analyst wishes depending on the amount of total variation desired.

For a pessimistic approach, the constraints above are then reformulated:

$$B_t = B_{t-1} + \sum_y (b_{t,y} - (BU_{t,y} \cdot k \cdot b_{t,y})) - \sum_{y,l} \left(pc_y D_{t,y,l} + rc_y Z_{t,y,l} \right) - \sum_{e,l} omc_e Q_{t,e,l}$$

$$\sum_{y,l} pc_y D_{t,y,l} \quad + \quad \sum_{y,l} rc_y Z_{t,y,l} \quad + \quad \sum_{e,l} omc_e Q_{t,e,l} \quad \le \quad \sum_{y} (b_{t,y} - (BU_{t,y} \cdot k \cdot b_{t,y}))$$

and the following constraints are added:

$$\sum_{t,y} BU_{t,y} \ge \Gamma$$

$$0 \le BU_{t,y} \le 1$$

This approach is pessimistic because the new budget constraints are tighter than in the baseline formulation. The RO variables $BU_{t,y}$ and constraints force a total reduction in the budget amount controlled by the budget of variability Γ.

For an optimistic approach, the robust counterpart can be formulated to allow for an overall increase in the budget levels as well as to allow transfers of funds from one system or time period to another. For this formulation, different RO variables are introduced, $B_{t,y,+}$ and $B_{t,y,-}$. There are also two budgets of variability: Γ_+ and Γ_-. The constraints from the baseline above are then reformulated:

$$B_t = B_{t-1} + \sum_{y} (b_{t,y} + ((BU_{t,y,+} - BU_{t,y,-}) \cdot k \cdot b_{t,y})) - \sum_{y,l} (pc_y D_{t,y,l} + rc_y Z_{t,y,l}) - \sum_{e,l} omc_e Q_{t,e,l}$$

$$\sum_{y,l} pc_y D_{t,y,l} + \sum_{y,l} rc_y Z_{t,y,l} + \sum_{e,l} omc_e Q_{t,e,l} \le \sum_{y} (b_{t,y} + ((BU_{t,y,+} - BU_{t,y,-}) \cdot k \cdot b_{t,y}))$$

To control the RO variables, the following constraints are added:

$$\sum_{t,y} BU_{t,y,-} \le \Gamma_-$$

$$\sum_{t,y} BU_{t,y,+} \le \sum_{t,y} BU_{t,y,-} + \Gamma_+$$

$$0 \leq BU_{t,y,+} \leq 1$$

$$0 \leq BU_{t,y,-} \leq 1$$

The RO variables work together to allow for an overall increase in the budget amounts (\$) across the systems and time periods, as well as allow for a transfer of funds among systems and time periods. For this formulation, a budget of variability for Γ_+ greater than zero allows for an increase in the total budget across the indices t and y. The budget of variability for Γ_- controls the amount of funds transfers, regardless of the value of Γ_+. This is considered an optimistic approach because its operational implication is that funds are flexible and can be transferred among systems and time periods to get the most utility.

3.3.2 Priorities

For the priority parameter for each mission, the robust counterpart formulation will be different than for the budget because the priority parameter p_m appears in the objective function as a scalar for each mission:

$$\max_{\substack{P, G, Z \\ D, S, X, \\ B, Q,}} z = \sum_{(t,m,c,i):t \in T(r),d_{t,m}>0,hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,c,i}}{\sum_{i'} hi_{m,i'}}$$

To create a robust counterpart, the RO variable PM_m replaces the parameter in the objective function:

$$\max_{\substack{P, G, Z \\ D, S, X, \\ B, Q, PM}} z = \sum_{(t,m,c,i):t \in T(r),d_{t,m}>0,hi_{m,i}>0} PM_m ok_{m,i,c} \frac{X_{t,m,c,i}}{\sum_{i'} hi_{m,i'}}$$

The following constraints are added to the formulation to control the value range that PM_m can take:

$$(1-k)p_m \leq PM_m \leq (1+k)p_m$$

$$\sum_m p_m = \sum_m PM_m$$

For each mission, the difference between the nominal value and the priority variable cannot be more than k percentage of the nominal value. By forcing the sums of the variables PM_m to equal the sum of the nominal parameters p_m the overall priority value does not change. From a conceptual point of view, this would be much more powerful and insightful if the priorities are normalized, and relative priorities of various missions can be intuitively compared since the priority of a mission only has meaning when compared to the priority of another.

This is an optimistic approach because it allows the solver to apply more priority value to those missions in the objective function that can accrue the most value. Operationally this would mean priority would be taken from hard-to-complete missions and given to easier-to-complete missions (since the mission values accrued are ratios of hours met to hours required) and thus that satisfying demands that are easier to meet would yield more operational (objective) value, a very optimistic outlook.

On face value, there is not a viable pessimistic alternative. The sum of the new priorities could be required to be less than the sum of the nominal values, but it would have the same operational outcome as keeping the sums the same. That is, the solver algorithm would still want to take priority value from hard to complete missions to easier to increase easier complete missions. Section 3.4 discusses these challenges more in depth and presents a method to develop a pessimistic robust counterpart formulation for the priority values. Other techniques could also be explored to develop pessimistic or worst-case scenarios, such as Benders' decomposition, that could be valuable future work (see Section 5.3).

3.3.3 Mission Demand

There are two approaches that can be taken for modeling uncertainty on the mission durations. The first is to consider the mission durations from the perspective of the number of time periods in which a mission occurs. The second approach is to consider the mission demands within a time step: the frequency of the mission (think sorties) and the hours required per sortie. Both of these are numeric parameters, and the product of them is the number of hours required for a mission in a demands. The hours (number of hours required for a system configuration to be on station to meet the mission) as a parameter expresses a total for the mission, as it accounts for hours of different intelligence types. Perhaps adding

a new intelligence type *uncertain* that can act as the scaling parameter for the mission durations would be useful. Either way, this is done in the preprocessors, as there is no constraint in the formulation that limits the total mission demands to the sum of the different types. The mission demands by type of intelligence are, as mentioned, accounted for separately in the formulation in the objective function and the constraints.

The second approach is attractive because it allows for some perturbations in the mission demands that obviously have an effect on the scheduling, and one can say that for any given mission demand that the SMEs are accurate within a certain percentage range about the nominal values for hours of mission demand.

Taking this approach, the robust counterpart formulation for the mission demands is similar to that taken for the mission priorities. The hours required for a mission are represented by the parameter $hi_{m,i}$. In the baseline JPAT formulation, this parameter appears in the objective function and in the constraints:

$$\max_{\substack{P,\,G,\,Z \\ D,\,S,\,X, \\ B,\,Q,}} z = \sum_{(t,m,c,i):t\in T(r),d_{t,m}>0,hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,c,i}}{\sum_{i'} hi_{m,i'}}$$

$$\sum_{c:ok_{m,i,c}>0} X_{t,m,c,i} \le hi_{m,i}d_{t,m}$$

To make the robust counterpart formulation for the hours of demand, the robust variable HI_{mi} is introduced, and replaces the parameter $hi_{m,i}$ in the about equations:

$$\max_{\substack{P,\,G,\,Z \\ D,\,S,\,X, \\ B,\,Q,\,HI}} z = \sum_{(t,m,c,i):t\in T(r),d_{t,m}>0,hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,c,i}}{\sum_{i'} HI_{m,i'}}$$

$$\sum_{c:ok_{m,i,c}>0} X_{t,m,c,i} \le HI_{m,i}d_{t,m}$$

As with the priority robust counterpart in section 3.3.2, the parameter k is introduced and the following constraints are added to the formulation:

$$(1-k)hi_{mi} \leq HI_{mi} \leq (1+k)hi_{mi}$$

$$\sum_{mi} hi_{mi} = \sum_{mi} HI_{mi}$$

These constraints allows each robust variable to range about a certain percentage of the nominal value, scalable by the parameter k. The final constraint added keeps the overall mission demand hours the same. This is an optimistic approach because the robust variables will increase or decrease from their nominal values where they will provide the most overall utility. Section 3.4 discusses challenges and solutions to creating a pessimistic approach or scenario to the robust counterpart for mission demands.

3.4   Pessimistic Formulation

The robust optimization formulation methods can be applied to the monthly budget parameters in such a way that allows the analyst to choose an optimistic or pessimistic approach. For example, one can allow the transfers of budgets among the time periods that can be optimally applied to accrue more utility. Conversely, the analyst can force an overall decrease in the budget parameters by a specified amount to create a pessimistic formulation.

The robust counterparts for the priority and mission demand hour parameters cannot be formulated in the same way as the budgets in the context of allowing pessimistic scenarios. In the case of the priorities, forcing an overall decrease in the budget of uncertainty on the parameter is effectively meaningless. The priorities for missions are weights in a weighted sum and only have meaning relative to each other. The demand hours parameter also appears in the objective function, and provides its own challenge to the formulation of a pessimistic scenario. The demand hour parameter is the denominator of the of the weighted sum's ratio. In this case it would be unclear what an interpretation of a pessimistic scenario would mean. A reduction in demand hours of a mission would increase the total utility value from the objective function. An increase in demand hours could also lead to an increase in total utility value as there is more potential value to accrue.

To overcome the conceptual and interpretation challenges of a naively formulated robust optimization application to the mission priority and demand hour parameters, a differ-

ent modeling approach needs to be taken. The problem is approached from an attacker-defender standpoint: The Army is "attacking" when assigning assets to accrue utility value by satisfying mission demands, and Uncertainty is "defending" when making those asset assignments the least valuable by minimizing the utility over the robust variables. This would be a min-max class of problem, and introduces its own modeling difficulties.

To solve most min-max problems, the preferred method would be to take the dual of the inner primal problem and then minimize over all the variables. This is preferred when conditions can be met that would assure strong duality as the optimal solutions of the dual and primal would then be equivalent. As a mixed-integer program, the JPAT formulation unfortunately does not meet all of the conditions of strong duality.

The approach taken below is to optimize the baseline JPAT and determine what the optimal variable assignment levels are (which is essentially the optimal plan). The decision variables are fixed at the levels for the optimal plan, and the robust counterpart is solved, minimizing over only the robust variables. This reveals the worst realization of the optimal plan subject to the variability of the robust parameters. The steps for this for the mission demands follow:

1. Solve the baseline JPAT for z^*:

$$\max_{P,G,Z,D,S,X,B,Q} z^* = \sum_{(t,m,c,i):t\in T(r),d_{t,m}>0.hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,i,c}}{\sum_{i'} hi_{m,i'}}$$

s.t.

$$\sum_{c:ok_{m,i,c}>0} X_{t,m,i,c} \leq hi_{m,i} d_{t,m} \quad \forall t \in T(r), e, l : t > 1$$

...

2. Fix all decision variables to current levels (P,G,Z,D,S,X,B,Q)
3. Solve the robust counterpart JPAT for z^* minimizing over the robust variables:

$$\min_{HI_{mi}} z^* = \sum_{(t,m,c,i):t\in T(r),d_{t,m}>0.hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,i,c}}{\sum_{i'} HI_{m,i'}}$$

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s.t.

$$\sum_{c:ok_{m,i,c}>0} X_{t,m,i,c} \leq HI_{m,i}d_{t,m} \quad \forall t \in T(r), e, l : t > 1$$

...

The case for the pessimistic robust counterpart to the mission priorities follows a similar sequence. In both cases the optimal plan is not allowed to change. The scenario does reveal how the utility of the optimal plan degrades subject to the variability on the robust parameters. This information may be of only marginal use as applied JPAT model. This is because the valuable output of the JPAT is the variable assignment levels and not the objective value itself. For a model in which the objective value is an important output, it can be very informative. For example, if the objective function was a measure of the flow of a commodity through a network, such as water through underground pipes, it would be good to know how your total flow is affected by some amount of variability of some parameter (ambient temperature, seismic activity, etc.), and whether or not the flow would fall below some critical threshold.

For the analysis conducted in Chapter 4, the robust counterparts for the mission priorities and mission demands are used as depicted above. It finds the worst-case outcome of a given baseline plan for a given level of variability (as defined by the variability parameters) while maintaining the plan's feasibility.

Though outside the scope of this thesis, it would be interesting to know what realizations or levels of variability of the robust variables would make the optimal plan infeasible. One method to explore this concept would be to introduce elastic variables for a given decision variable and penalize it in the objective function. The elastic variables would allow the model to remain feasible while revealing the infeasibility of the solution in an operational context. This would be an interesting topic for future research.

$$\max_{\substack{P,\,G,\,Z \\ D,\,S,\,X, \\ B,\,Q,}} \quad z = \sum_{(t,m,c,i):t\in T(r),d_{t,m}>0,hi_{m,i}>0} p_m ok_{m,i,c} \frac{X_{t,m,c,i}}{\sum_{i'} hi_{m,i'}} \tag{1}$$

$$\text{s.t.} \quad \sum_{c:ok_{m,i,c}>0} X_{t,m,c,i} \le hi_{m,i} d_{t,m} \qquad \forall\, t \in T(r), m, i : d_{t,m} > 0, hi_{m,i} > 0 \tag{2}$$

$$Q_{t,e,l} = Q_{t-1,e,l} + es_{y,e} \sum_y \left(D_{t,y,l} - Z_{t,y,l} + \sum_{l'} (G_{t,y,l',l} - G_{t,y,l,l'}) \right) \qquad \forall\, t \in T(r), e, l : t > 1 \tag{3}$$

$$Q_{t=1,e,l} = iq_{e,l} \qquad \forall\, e, l \tag{4}$$

$$P_{t,c,l} \le \frac{Q_{t,e,l}}{ec_{c,e}} \qquad \forall\, t \in T(r), l, c, e : ec_{c,e} > 0, \exists m \in M(l) : d_{t,m} > 0 \tag{5}$$

$$S_{t,m,c} \le sr_{m,c} d_{t,m} P_{t,c,l(m)} \qquad \forall\, t \in T(r), m, c \tag{6}$$

$$X_{t,m,c,i} \le \frac{hm_m S_{t,m,c}}{sr_{m,c}} \qquad \forall\, t \in T(r), m, c, i : ok_{m,i,c} > 0, hm_{m,i} > 0, d_{t,m} > 0 \tag{7}$$

$$\sum_{y,l'} ht_{e,y,l,l'} G_{t,y,l,l'} + \sum_{c,m\in M(l)} ec_{c,e} \left(\frac{hm_m}{sr_{m,c}} + su_e \right) S_{t,m,c} \le he_e Q_{t,e,l} \qquad \forall\, t, e, l \tag{8}$$

$$B_t = B_{t-1} + \sum_y b_{t,y} - \sum_{y,l} \left(pc_y D_{t,y,l} + rc_y Z_{t,y,l} \right) - \sum_{e,l} omc_e Q_{t,e,l} \qquad \forall\, t \in T(r) \setminus N : t > 1 \tag{9}$$

$$\sum_{y,l} pc_y D_{t,y,l} + \sum_{y,l} rc_y Z_{t,y,l} + \sum_{e,l} omc_e Q_{t,e,l} \le \sum_y b_{t,y} \qquad \forall\, t \in T(r) \cap N \tag{10}$$

$$B_t = 0 \qquad \forall\, t \in T(r) \cap N \tag{11}$$

$$\sum_{l,t'\le t} D_{t',y,l} \le max_{t,y} \qquad \forall\, t \in T(r), y \tag{12}$$

$$\sum_{t'\le t,y:(y,y')\in REP} Z_{t',y,l} \ge \sum_{t'\le t} D_{t',y',l} \qquad \forall\, t \in T(r), l, y' : \exists y : (y,y') \in REP \tag{13}$$

$$P_{t,c,l} \in \{0,1\} \qquad \forall\, t \in T(r), c, l \tag{14}$$

$$G_{t,y,l,l'} \in \mathbb{Z}^+ \qquad \forall\, (t,y,l,l') \in GP : t \in T(r) \tag{15}$$

$$Z_{t,y,l} \in \mathbb{Z}^+ \qquad \forall\, t \in T(r), y, l \tag{16}$$

$$D_{t,y,l} \in \mathbb{Z}^+ \qquad \forall\, t \in T(r), y, l \tag{17}$$

$$X_{t,m,c,i} \ge 0 \qquad \forall\, t \in T(r), m, c, i \tag{18}$$

$$S_{t,m,c} \ge 0 \qquad \forall\, t \in T(r), m, c \tag{19}$$

$$Q_{t,e,l} \ge 0 \qquad \forall\, t \in T(r), e, l \tag{20}$$

$$B_t \ge 0 \qquad \forall\, t \in T(r) \tag{21}$$

Figure 3.1: Mathematical formulation for JPAT

CHAPTER 4:

Analysis

Section 4.1 discusses the experiments and test data used. Section 4.2 presents the results and analysis of the experiments conducted.

4.1 Experiments

A test data set was constructed to provide a realistic, plausible scenario in which to run experiments on the JPAT robust counterpart formulations. It is of a smaller scale in terms of number of parameters and variables than the actual JPAT data that resides in the classified operating environment.

Six robust counterpart models of the baseline JPAT formulation were created, as described in Chapter 3. Experiments were conducted to explore the behavior of the models under different levels of induced variability. Of interest is the impact of the variability parameter k and the budget of variability Γ on the objective values and solutions to the optimization models. The experiments provide insights to analysts and decision makers about the operational and modeling impacts when using robust optimization techniques to model variability and uncertainty.

4.2 Numerical Results

In the first experiment, 101 iterations of the optimistic robust budget scenario were run, incrementing the parameter k from 0 to 1 by 0.01 each time. In each run, the value of k represents the percentage away from the nominal values of the budgets $b_{t,y}$ that the robust variables could take. In Figure 4.1 the objective value (referenced by Z from here out) is plotted for each run against its k. Z is increasing in k, and generally concave. Note also that the right vertical axis of the graph shows the percentage difference from the baseline JPAT objective value for this test data ($Z_{BASELINE} = 6.52$).

Figure 4.1 shows a continuous increase in the value of Z in the range of about $k = 0.10$ to $k = 0.30$. This region is interesting and worth exploring from both an analyst perspective as well as an operational perspective. For any given point estimate given by a SME, the
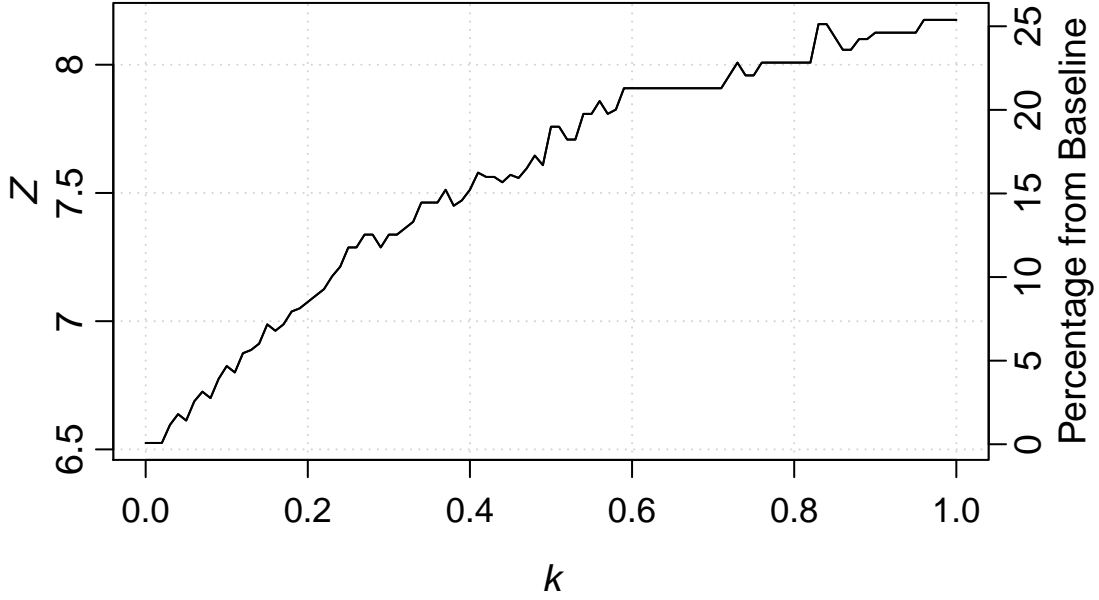
RO Formulation: Budget, Optimistic

Figure 4.1: Objective value of the robust formulation for optimistic budgets as a function of k. The parameter k in the formulation controls the percentage about the nominal value of the budget parameter $b_{t,y}$ that the robust budget variables $BU_{t,y}$ are allowed to vary.

true value due to variability (though highly dependent on the subject matter and the SME expertise) would reasonably be within 10 to 30 percent of the estimate. Exploration of the operational impacts (mission assignments, procurement schedules, etc.) that occur around these ranges of variability can provide key insights to mitigating operational risk.

Using the test data, the budget expended when $k = 0.10$ is \$168M and when $k = 0.30$ is \$197M. This difference of \$29M reduces the number of unmet mission demands by about one quarter and leads to a significant jump in utility. For a decision maker, this difference in the objective value for a given input (such as operating budget) also yields the optimal assignment of that input when using a robust formulation. The robust formulation could also reveal any large jumps in the objective value over a small range of k, and knowing that these "cliffs" exist on the objective value function is useful because mitigating steps may
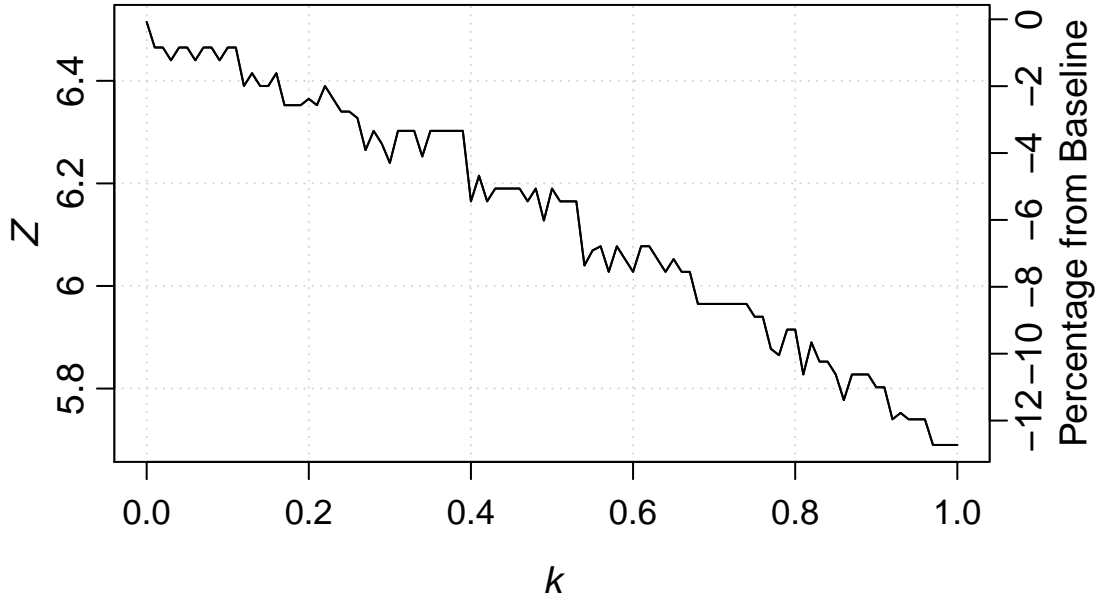
RO Formulation: Budget, Pessimistic

Figure 4.2: Objective value of the robust formulation for pessimistic budgets as a function of *k*

then be taken to overcome that risk frontier if they are operating near the edge of it.

Similar experiments were conducted on the pessimistic robust formulation for the budget parameters: 101 iterations of the optimization for each incrementation of the parameter k from 0 to 1. The objective value Z as a function of k is non-convex in this scenario as well, but it does not have the plateau evident in the optimistic formulation. This is because each iteration of the model has tighter constraints than the one before it, so an incumbent solution is not at all likely to remain from one iteration to the next (see Figure 4.2). However, there appear to be "neighborhoods" of solutions such that a given range of the parameter k yields a tight range of similar objective values and solutions. An analyst can examine a candidate range of k values to glean insights into operational impact similar to the optimistic scenario. Examination of the frontiers of the likely variability region can provide useful insights to models and problems beyond the scope of the JPAT formulation.

The same analyses of the *k* parameter were conducted on the robust counterpart for the priority parameter. For the optimistic scenario, the objective value increases in *k* in a roughly linearly manner (see Figure 4.3). These results are what is expected. Unlike in the budget robust counterpart though, there are no regions where a slight perturbation in the uncertainty can have an outsize influence on the objective value. Close examination of the decision variable levels reveals that there is no significant impact from an operational perspective regarding system procurement and assignment. For this test data the priority parameters do not have an operational impact beyond the utility revealed by the objective function. Follow on research could reveal under what conditions variability on the weights in a weighted sum (as the priority parameters are) could impact the basis of a model solution.
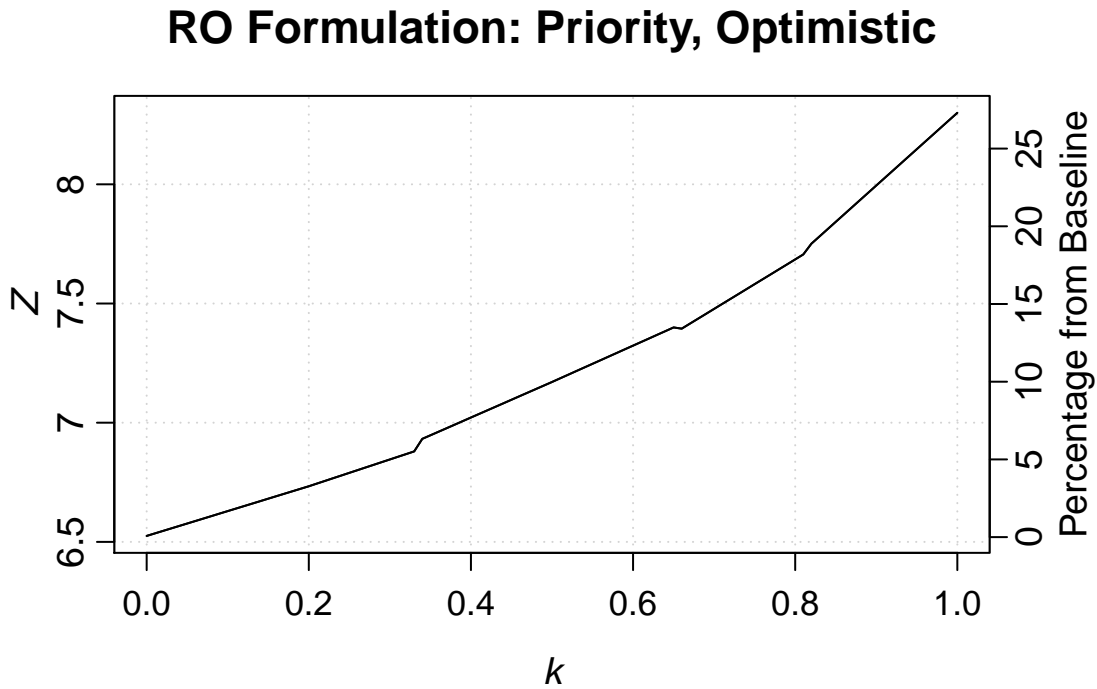
RO Formulation: Priority, Optimistic

Figure 4.3: Objective value of the robust formulation for optimistic priorities as a function of *k*.

The same results hold for the pessimistic case of the priority parameters because of the method used to construct the pessimistic scenario. The bisection search method can find

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the worst case outcome for a given variable assignment solution and allowed amount of variability. Figure 4.4 shows the linear effect of k on the objective value.
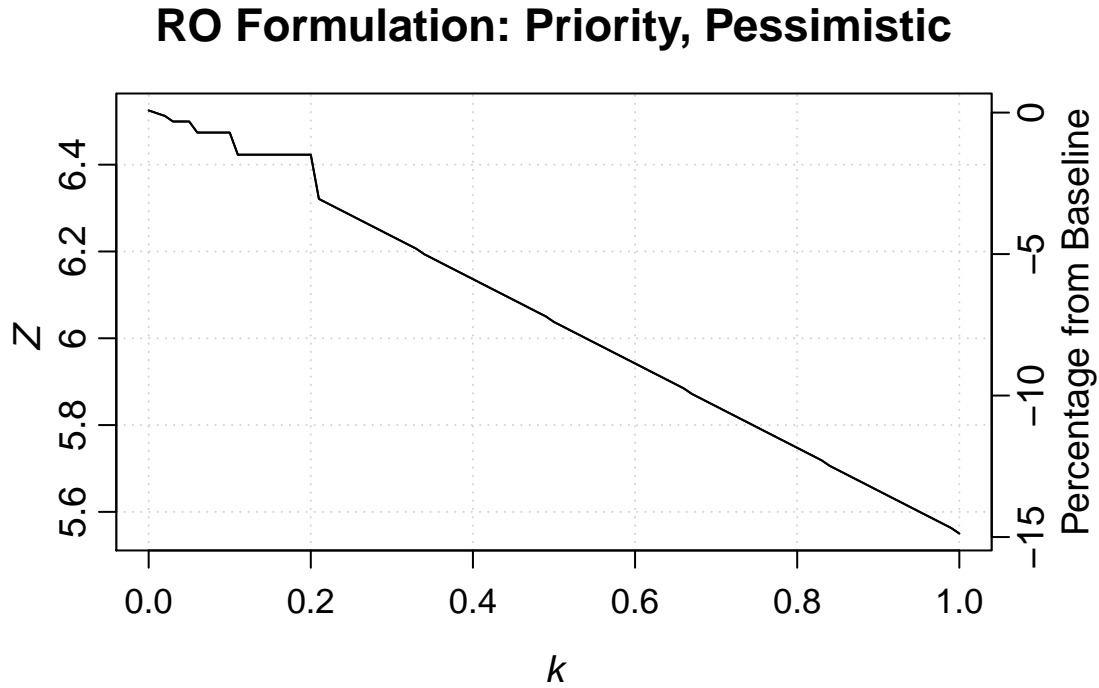
RO Formulation: Priority, Pessimistic

Figure 4.4: Objective value of the robust formulation for pessimistic priorities as a function of k.

Analysis of the k parameter on the on the robust counterparts for the mission demand parameters reveal similar behavior as that for the priority parameter. Figures 4.5 and 4.6 for the optimistic and pessimistic scenarios reflect the expected impact on the objective functions (increasing and decreasing in k, respectively, and). There are erratic behaviors at the extreme values of k ($k < 0.1$ for pessimistic, and $k > 0.9$ for optimistic) reflecting the solvers attempts at integer solutions. Figure 4.6 also shows that even in extreme cases of variability, the worst outcome of the optimization is no less than 6 percent worse than the baseline. The ability to reveal behavior such as this using a robust formulation is useful, and can allow decision makers to focus their efforts and resources on mitigating scenarios that produce worse outcomes.

For both the priority and demand robust counterparts, the parameters under variability ap-
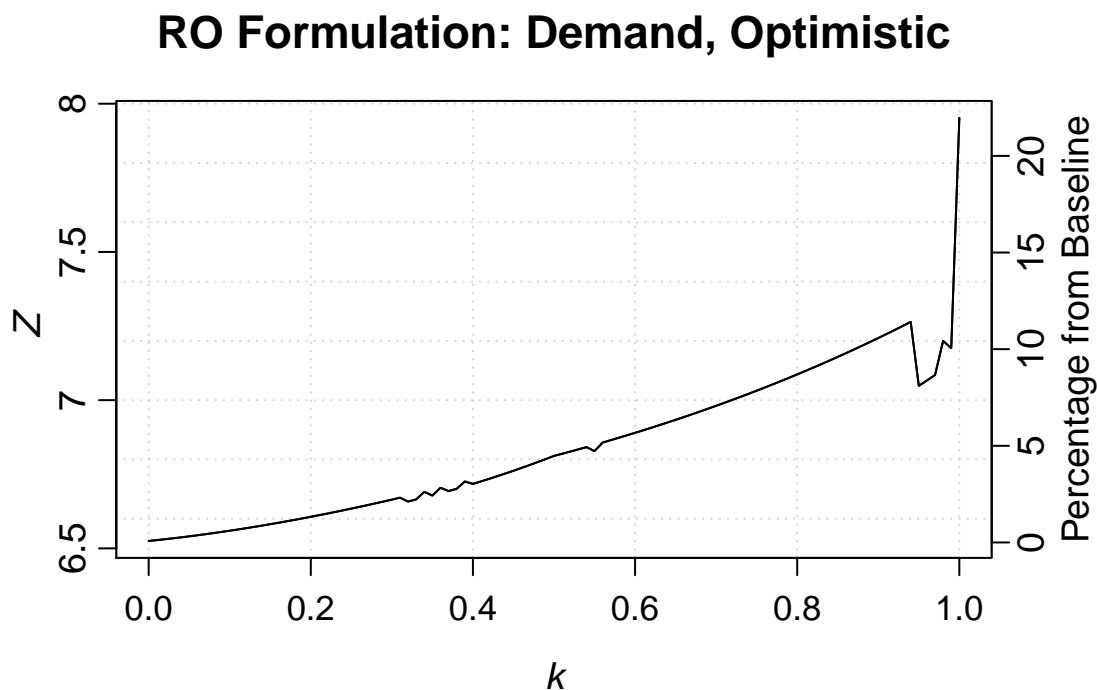
RO Formulation: Demand, Optimistic

Figure 4.5: Objective value of the robust formulation for optimistic demands as a function of *k*.

pear in the objective function. For the JPAT model and this test data, the introduction of variability in the objective function yields mundane results. This is partly due to the fact that for a problem such as the JPAT, the objective value itself is not a primary outcome of the model, nor is it a very informative piece of information. The value of the JPAT is in the variable assignments: what systems to purchase when, which machines to assign where.

For models similar to the JPAT, variability introduced on the constraints can reveal those areas of risk and alternative variable assignment levels that are interesting from an operational perspective. Likewise, variability can be modeled robustly in those areas that have the most impact, and to a degree that fits the users needs and reflects the operating environment. For example, variability parameters such as *k* don't have to be uniform across an entire model. They can be applied in a robust formulation in a way that allows different parameters to have different ranges of variability about their point estimates. This can often be the case for formulations that model problems over time: The budget in five years
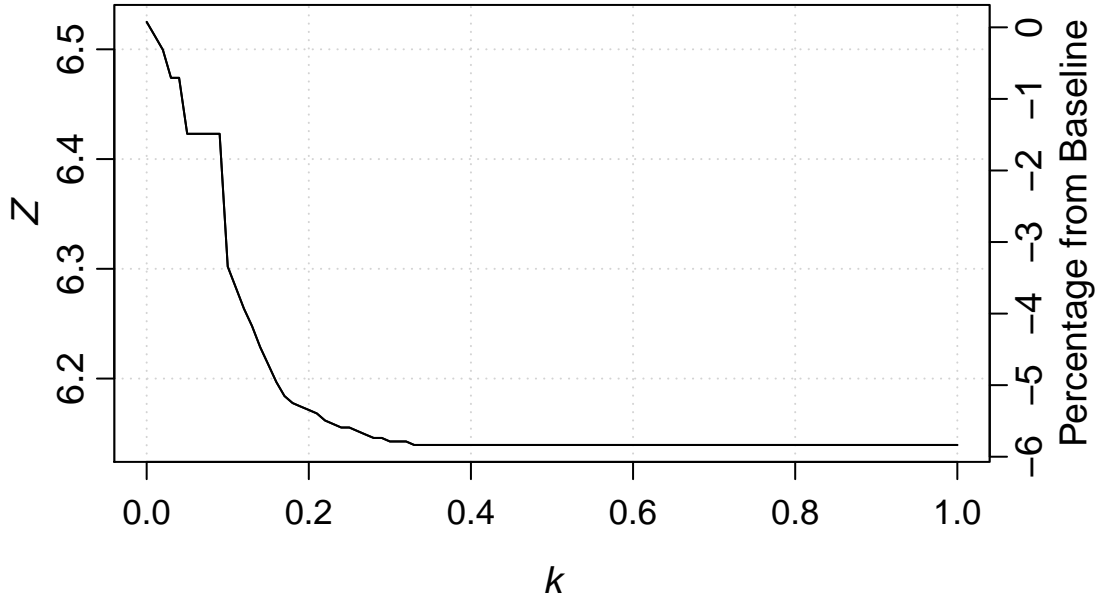
RO Formulation: Demand, Pessimistic

Figure 4.6: Objective value of the robust formulation for pessimistic demands as a function of *k*.

known with a lesser degree of precision than the budget for next year. Also, variability can appear on different parameters within a single model, with each their own of variability constraints or all subject to common ones.

Experiments were conducted to explore the interaction of the variability parameter k and the budget of variability Γ. The optimistic robust counterpart for the budget parameters was executed iteratively over a sample of chosen k sizes, from 0 to 0.5, and for each model was optimized with incremented values of Γ from 0 to 55. Figure 4.7 shows that with a higher variability parameter of k, the value of Γ has a continual impact on the solution as it scales up. Likewise, for a lower value of k, the value of Γ has less influence on the solution. In essence, for the optimistic scenario, the more variability about the nominal values (higher k values), the more of an impact that Γ (variability budget parameter) has on the objective value. If k is low, a greater increase in Γ makes little difference because variability on each applicable parameter would be maximized before the variability constraint of Γ is tight.
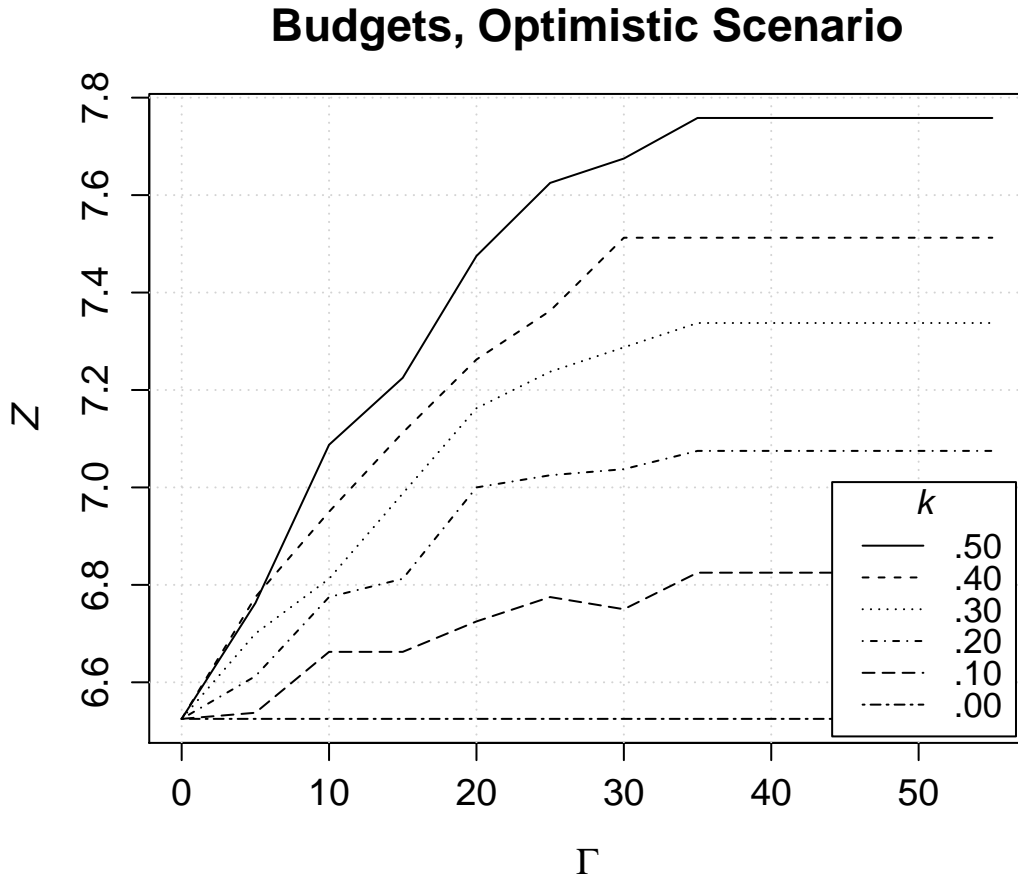
Figure 4.7: Objective value of the robust formulation for optimistic budgets as a function of Γ for various k values.

Interaction of the variability parameter k and the budget of variability Γ in the pessimistic scenario is examined in Figure 4.8. In this case the budget of variability is always tight in the constraints. If k is relatively high, such as $k = 0.5$, then as Γ increases, the objective value and decision variable solutions continually change. The value of Γ also has a greater impact on the objective value if k is high rather than low (such as $k = 0.1$). As a modeling consideration, it is important to realize that a value of Γ that is chosen too high will make the formulation infeasible. This is reflected in Figure 4.8: Regardless of the value of k, all the models become infeasible at values of $\Gamma \geq 40$.
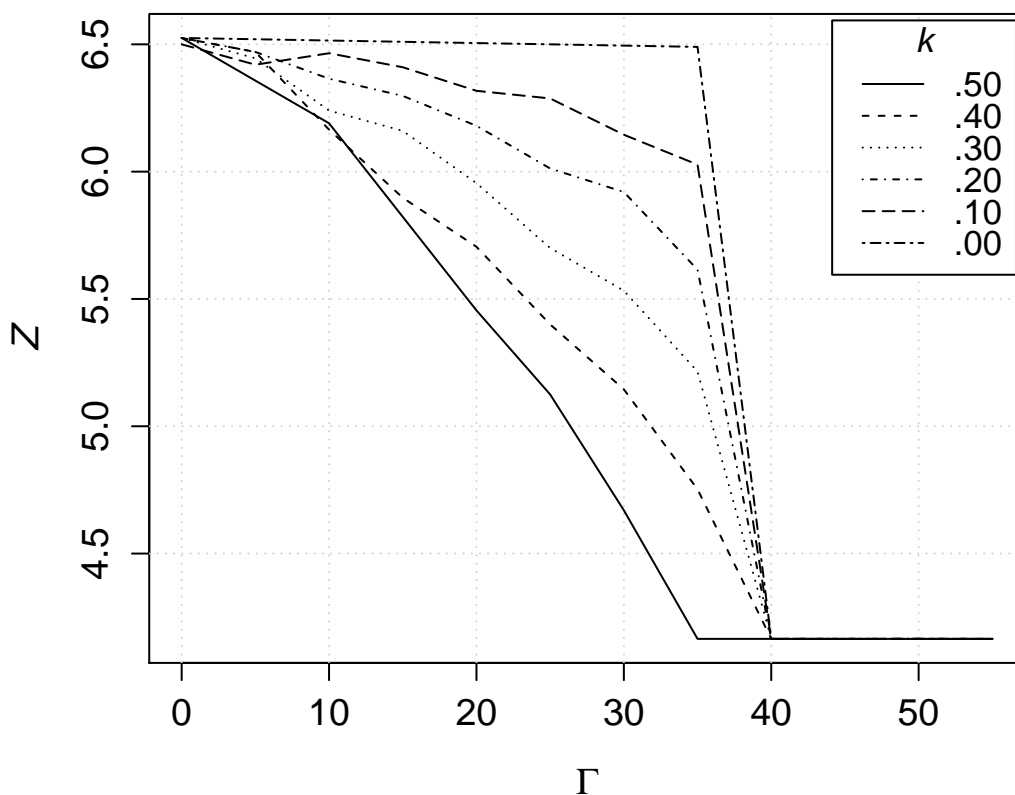
Budgets, Pessimistic Scenario

Figure 4.8: Objective value of the robust formulation for pessimistic budgets as a function of Γ for various k values.

Understanding of this dynamic between these two parameters allows analysts to "right-size" the parameters for a robust optimization model to fit the needs of the problem or model under examination. For example, in a pessimistic scenario, by allowing high variability with k and a low Γ, the analyst can identify which points of the model are most vulnerable to high uncertainty.

In the robust counterparts for the priority and mission demand parameters, there is no Γ parameter to explore, and thus no experiment to examine it. In the case of mission priority it is meaningless to allow overall priorities to increase or decrease: they only have value

relative to each other. For the hours of mission demand, a logical interpretation is hard to arrive at for allowing an overall increase or decrease in terms of which is better or worse (or optimistic or pessimistic) because the objective function is a sum of ratios of hours of demand met.

CHAPTER 5:
Results and Conclusions

Section 5.1 summarizes the results from Chapter 4. Section 5.2 discusses the conclusions and lessons learned from this research. Possible future research areas that can build on this thesis research are in section 5.3.

5.1 Results

The first set of experiments examined the effect of the robust parameter k on the objective value. The results were as expected. For each of the robust counterparts, when $k = 0$, the objective value and variable assignments are the same as for the baseline JPAT formulation for the test data set. The objective value increases as k increases for the optimistic counterparts, and decreases when k increases for the pessimistic counterparts, and the graphs of each appear generally concave as expected (see Figures 4.1 and 4.2 for example). The regions of the values of k in which the general slope of this concave function is higher reveal those areas of the model with a high amount of sensitivity to the robust parameters. Likewise, those areas with a lower slope are less sensitive to variability.

Depending on the primary output of the model in question, the value of k can provide useful insights to the decision maker. For the JPAT model, the primary outputs are the variable assignments (UAVs assigned to missions, transfers between locations, procurements and retirements, etc.) that drive the objective value. The robust counterparts for the budgets provide information on where to optimally expend those budget resources, a key interest of any decision maker.

The experiments on the effect of the robust parameter Γ reveal an interaction with k. In an optimistic scenario, if k is low, a greater increase in Γ makes little difference because variability on each applicable parameter would be maximized before the variability constraint of Γ is tight. Likewise, for higher k values, the value of Γ has a greater impact on the solution to the model. For the pessimistic scenario, the parameter Γ is always tight in the constraints.

5.2 Lessons Learned

This thesis focuses on one tool in the Army's analysis arsenal. However, all military decision making involves some level of uncertainty and risk. The insights derived from this research can be broadly applied to most optimization models involved in military operations analysis. Most analysts consider the inputs to such models to be point estimates. These inputs may model a range of possibilities and be presented as an average, or try to represent a worst case scenario, but almost always are represented by a single value. To avoid the pitfalls associated with this approach, model inputs should be represented as a range of values where it is appropriate to do so. Robust optimization techniques offer one method to do so.

Extensive adoption of such robust modeling techniques would benefit the Army analysis community by giving analysts a method to consider variability on the model during the formulation process and consider the operational impact. Oftentimes, the effect of uncertainty on a model is considered only after it is formulated, when sensitivity analysis is conducted. The problem with this approach is that the model may not lend itself to easy examination of uncertainty. For example, if an analyst wanted to run the JPAT baseline formulation to explore a 10 percent decrease of the overall budget, she would have to choose which budget values to decrease across all the time periods and systems under consideration (or naively decrease each budget parameter by 10 percent). Each choice of how that 10 percent cut is imposed could potentially lead to a unique optimal solution. But if she where to run the robust counterpart of the JPAT, she could set the levels of variability parameters to impose an overall 10 percent budget cut, and the solver would find the optimal solution and the associated budget amounts, indicating where and by how much the cuts should be made to yield the most utility for the end-user.

The range of variability under which a parameter should be examined will almost always be driven by real-world circumstances. In keeping with the previous example, perhaps the funding levels of the Army's aerial ISR programs are projected to be cut between 10 and 25 percent for the next fiscal year. These cuts drive the values of the variability parameters as chosen by the analyst, and defines a variability frontier that is of extreme interest to decision makers. Examination of these frontiers on the operational impact of these budget cuts can help decision makers mitigate the impact by optimally allocating resources. Being able to

quantify the impact of such budget decisions can help frame an argument for advocating for additional resources. Examination of the likely variability frontiers can also reveal those model solutions that are highly sensitive to uncertainty, where a small shift in an input parameter can have a large impact on model solution or objective function. Adjusting the variability parameters can identify those frontiers on the objective function hyperplane.

The scale and size of variability parameters (k and Γ used in the JPAT robust counterparts) are important modeling considerations. For example, in the pessimistic budget scenario a very high Γ could make the model infeasible, or extreme values of k and Γ may be completely unrealistic. The parameter values chosen and how variability is introduced to a model can be quite flexible. Variability can be modeled in unique ways for any given parameter within the same model. The ranges about the nominal value can also vary over indices (for example, a greater range of uncertainty further in the future, a lesser range sooner in the future). The robust variables can be restricted by unique budgets of variability (Γ) for each or from a common one. Scenarios can be modeled, such as best- and worst-case, that can contribute to course of action analysis. This allows the analyst to compare the model solutions (levels of the decision variables, for example) among different scenarios and know how robust a particular solution set is subject to the variability under consideration. The robust optimization techniques explored here provide an analyst a flexible tool set to examine optimization under uncertainty.

Depending on the underlying model, robust counterparts generally keep the same same class of of problem as their baseline. For most such models, the computational cost is negligible when contrasted with alternatives such as stochastic programming.

The robust optimization methodologies and the formulation techniques developed here and applied to the JPAT may be applied to many other types and classes of problems. Doing so provides a direct and computationally efficient method of accounting for risk and better informing military decision making.

5.3   Future Research Possibilities

This research has identified several areas of future work that may prove fruitful:

1. Comprehensive examination of variability on weights in an objective function,

specifically what conditions could lead to a meaningful impact on a model's solution set.

2. Research into methods of modeling variability on parameters that have known distributions, or apply scaling weights to variable parameters dependent on their distance from their nominal values.

3. Apply attacker/defender techniques on worst-case scenario generation for robust counterparts for linear, continuous programs.

4. Apply elastic variables to robust counterparts to reveal and explore infeasibilities induced by the robust variable settings.

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